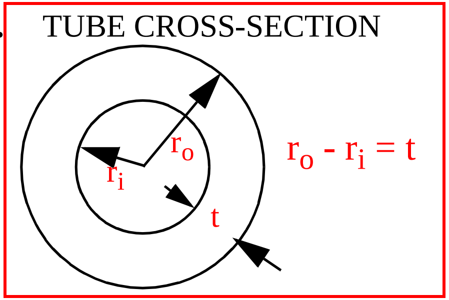
AOE/ESM 4084 "Engineering Design Optimization"

- Design variables (set of variables that describe the system):
  - $\mathbf{x} = x_1, x_2, \ldots\ldots, x_n$; or $\mathbf{x} = (x_1, x_2, \ldots\ldots\ldots, x_n)^T$
  - types of design variables (DV's)

    physical nature: dimensional, shape, material, topology

    numerical nature: continuous, discrete valued, integer, binary (0/1)
  - Selection of DV's may not be unique.
  - DV's should be independent of each other.
  - For design variables use as many independent parameters as possible.



- Objective function (merit function)

  measure of goodness of the system that is being designed:

  - $f(\mathbf{x}) = f(x_1, x_2, \ldots\ldots\ldots, x_n)$
  - to be Maximized or Minimized
  - types of objective function

  physical nature: cost, profit, weight, comfort, deformations, stress

  mathematical nature: linear or nonlinear

LINEAR

$$f(\boldsymbol{x}) = c_1x_1 + c_2x_2 + \ldots + c_nx_n = \begin{bmatrix} c_1 & \ldots & c_n \end{bmatrix}^T \begin{bmatrix} x_1 \\ \ldots \\ x_n \end{bmatrix} = \boldsymbol{c}^T\boldsymbol{x}$$

NON-LINEAR

$$f(\boldsymbol{x}) = c_1x_1^3 + c_2\cos x_2$$

- Constraints: define the boundaries of the feasible design space
  - constraints on design variables
    - direct limitations on DV's (side constraints); $x_i^L \le x_i \le x_i^U$ where $i = 1, \ldots, n$
    - relations between dv's (variable linking); e.g., $3x_1 + x_2 = x_3$
  - constraints on system behavior
    - limits on system output; e.g. stress, displacement, buckling load limits.
    - physical laws governing the system; conservation of mass, energy, momt.
  - inequality constraints

$$g_j(\boldsymbol{x}) \le 0 \quad j = 1, \ldots\ldots, n_g$$

  - equality constraints

$$h_k(\boldsymbol{x}) = 0 \quad k = 1, \ldots\ldots, n_e$$

- Formulation of an optimum design problem involves transcribing a verbal statement of the problem into a well defined mathematical statement

- Standard Mathematical Statement
  - ***Minimize***

$$f(\boldsymbol{x}) = f(x_1, x_2, \ldots\ldots, x_n)$$

$$\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T \qquad \boldsymbol{x} = \{x_1, x_2, \ldots, x_n\}$$

  - ***subject to***

$$g_j(\boldsymbol{x}) \le 0 \qquad j = 1, \ldots\ldots, n_g$$

$$h_k(\boldsymbol{x}) = 0 \qquad k = 1, \ldots\ldots, n_e$$

$$x_i^L \le x_i \le x_i^U \qquad i = 1, \ldots\ldots, n$$

Special Cases

- ***Problem is to maximize a function instead of minimization,***

  $$\textit{Max } f(\mathbf{x}) = \textit{Min } -f(\mathbf{x})$$

  $$\textit{Max } f(\mathbf{x}) = \textit{Min } 1/f(\mathbf{x})$$

- ***Optimizer cannot handle equality constraints,***

  $h(\mathbf{x}) = 0$ $\qquad g_1(\mathbf{x}) = h(\mathbf{x}) \leq 0$ *and* $g_2(\mathbf{x}) = -h(\mathbf{x}) \leq 0$

- ***Optimizer connot handle design variables with negative values,***

  say, $-a \leq x \leq b$ *where* a and b *are* +’*ve numbers*

  - *Introduce a new variable* $x' = x + a$ *where* $0 \leq x'$
  - *Introduce two new variables* $x^+ - x^- = x$